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On The Dynamics of People's Unions

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A new mechanism governing the dynamics of territory changing between groups of people such as countries is proposed based on trading with approval of both sides under particular voting rule (veto rule, majority rule, etc.). Conquest of the territory is viewed as a special case of trading. One- and two-dimensional cases are considered, where in the latter case the cost of border between countries is proportional to its length. A state equation is obtained based on particular personal utility function. Under migration rules maximal territory expansion and minimal possible territory are evaluated for small group surrounded by a big one. Parallels with statistical physics processes are revealed.

Key words: public economics, allocation theory, size of nations

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Предлагается новый механизм передачи территории между группами людей путем торговли с одобрения обоими сторонами посредством голосования по определённым правилам (правило простого большинства, правило вето, и т.п.). Завоевание территории рассматривается как частный случай торговли. Рассмотрены одномерные и двухмерные случаи, в последнем стоимость содержания границы пропорциональна её длине. Из функции полезности индивидуума получено уравнение состояния группы людей. При определенных правилах миграции проведена оценка радиусов минимально и максимально возможных площадей, занимаемых небольшой группой людей, в окружении другой большой группы. Приведены аналогии с процессами в физике.

Ключевые слова: экономика общественного сектора, теория размещения, размер стран

ISBN

Содержание

Introduction

This work studies the processes of origination and development of people's groups which own a particular territory. Not only countries could be an example of such groups, but also the municipalities of different levels and every group of land owners which members are not allowed to sell its land without permission of the group.

The historical examples could be taken from the works by Lev Gumilev (e.g., Gumilev, 1990) where he determined the concept of *ethnos* as a group of people having common model of behavior allowing them to mark out themselves from the others. Gumilev showed on the historical examples that people in the ethnos not necessarily have the same ethnicity, nationality, religion, language and so on. Gumilev thought that every ethnos has the particular stages of its development which are determined by the number of *passionary* persons it contain. He states *passionarity* as a feature of a person's temper to prefer the unusual (for the ordinary peoples) values such as glory, power and so on. In the given study we will try to give an economical basis for these concepts.

In the classical paper by Alberto Alesina and Enrico Spolaore (Alesina & Spolaore, 1997) the size and number of nations on the continuum of uniformly distributed individuals is considered particulary as an equilibrium where each individual at the border between two countries can choose which country to join with her land. There have been considered the coalition equilibrium as well as in (Bolton & Roland, 1997). Charles M. Tiebout in his paper (Tiebout, 1956) considered equilibrium where the consumer-voter is fully mobile and will move to one of the fixed number of communities where her preference pattern is best satisfied. The results of this paper was criticized by Bewley (Bewley, 1981).

In contrast with the just mentioned papers in the work in hand there is no free will of agent supposed. An individual can leave her country only with permission of other citizens and she can join the other country if its citizens agree with it. The second assumption is that the utilities of all agents are transferable so that individual can pay for permission to leave and to join the country. Thus, countries can trade their territory, a historical example of which is the buying Alaska from Russia by the United States of America in 1867. Stability of the result of the trade depends on the form of the utility functions and on the voting rules in the countries. The war is considered as a mechanism preventing cheating during the trade. The countries which have approximately the same marginal utility of the territory at the border would fight rather than trade as a result of equilibrium in the corresponding game. We also study two regimes of the group's size dynamics, where maximal and minimal territory of the group could be found. In this problem the parallels with processes of bubbling of the overheated liquid could be revealed.

This work gives the microeconomic basis for some of author's results in (Belyakov, 2007).

1 Mechanisms of border moving by trade

Let us consider two countries located on a unit interval (fig. 1) with territories S_1, S_2, and with transferable quazilinear social utility functions W_1, W_2. Territory could be exchanged

Рис. 1: Two nations on the interval

for money with approval by voting in both contries. Decision could be maid by *veto rule* when every citizen ought to approve it or by *majority rule* when more than half of the citizens should vote for it.

For simplicity we will further assume positive and monotonically decreasing marginal social utility of the country with respect to its territory.

1.1 Veto rule

When every citizen of the both countries ought to approve the deal, the Pareto efficiency is obtained. Since each person should be paid in order to compensate her disutility and social utility functions are quazilinear the problem of finding an equilibrium border positions is equivalent to the problem of finding extremums of sum social utility functions of both countries $W_1(x) + W_2(x)$. Stable border positions correspond to the maximum points

$$W_1(x) + W_2(x) \to \max_{x \in [0,1]} .$$

Here the first order condition (FOC)

$$\text{FOC (stationarity):} \quad \frac{\partial}{\partial x}(W_1(x) + W_2(1-x)) = 0 \tag{1}$$

is the condition of equilibrium, while the second order condition (SOC)

$$\text{SOC (stability):} \quad \frac{\partial^2}{\partial x^2}(W_1(x) + W_2(1-x)) \leq 0 \tag{2}$$

is the condition of stability to small perturbations of the border position.

If both countries have decreasing marginal utilities to their territories, there exists the unique internal solution for the equation (1) and inequality (2), i. e. a stable border position which guaranties them a Pareto optimum.

1.2 Majority rule

If approval of only one half of a given country citizens is sufficient for selling a part of its territory, buyer could pay lower price in comparison with the veto rule case. Let us suppose that country 1 pays an additional surplus of all its citizens to the ruling half of country 2 citizens. Such deals can be done while $\frac{\partial W_1(x)}{\partial x} \geq -\frac{1}{2}\frac{\partial W_2(1-x)}{\partial x}$. Hence a maximum $\overline{x}$ of the territory S_1 satisfies the condition:

$$\overline{x}: \quad \frac{\partial}{\partial x}(W_1(\bar{x}) + W_2(1-\bar{x})/2) = 0 \tag{3}$$

The same trick could be made by country 2 if it byes territory from country 1. So the lowest possible territory S_1 is $\underline{x}$ obtaining from the condition:

$$\underline{x}: \quad \frac{\partial}{\partial x}(W_1(\underline{x})/2 + W_2(1-\underline{x})) = 0 \tag{4}$$

So there could be an interval $[\underline{x}, \overline{x}]$ where border is possible but any point of this interval is not an equilibrium.

Proposition 1. With assumption of monotonically decreasing marginal utilities of the countries, this interval is not empty and $\underline{x} < \overline{x}$.

This result can easily be generalized on the case when a particular percentage of citizens

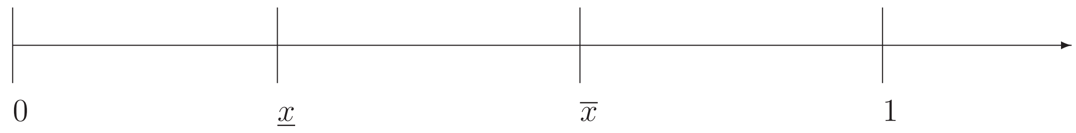

Рис. 2: Possible positions of the border under majority rule

make a decision about selling territory. Obviously the lower this percentage is the lower is the minimum possible territory of the country.

2 Multiple steady equilibria

We will give two examples where the steady equilibrium position of the border is not unique.

Let us at first rewrite FOC (1) and SOC (2) in terms of marginal welfares

$$p_1 = \frac{\partial W_1(S_1)}{\partial S_1}, \quad p_2 = \frac{\partial W_1(S_2)}{\partial S_2}, \tag{5}$$

where $S_1 = x$ and $S_2 = 1 - x$. Thus stable equilibrium border should satisfy

$$\text{FOC (stationarity):} \quad p_1 = p_2, \tag{6}$$

$$\text{SOC (stability):} \quad \frac{\partial p_1}{\partial S_1} + \frac{\partial p_2}{\partial S_2} \leq 0. \tag{7}$$

Variables p_1 and p_2 are the maximum (minimum) prices that countries 1 and 2 could pay (take) for increasing (decreasing) their territory by one unit.

If welfare functions are not continuously differentiable it might happen that its left derivation p^- is not equal to its right derivation p^+, specifically $p^- > p^+$ which is consistent with decreasing marginal utility required for stability. Then equilibrium condition (6) has the following view

$$\begin{cases} p_1^- \geq p_2^+, \\ p_1^+ \leq p_2^-. \end{cases}$$

Left (right) border of the continuum of equilibria could be obtained from the point where first (second) inequality becomes the equality.

2.1 Presence of transaction costs

Suppose that there are marginal costs of the new territory accommodation $t^+ > 0$ and of the territory selling $t^- \geq 0$. Then the left and the right derivations of welfare with respect to territory have the view

$$p^+ = p - t^+, \quad p^- = p + t^-.$$

Obviously $p^- > p^+$ which means that all points in the equilibrium interval are stable.

2.2 Behavioral effect

Prospect Theory by Kahneman and Tversky (Kahneman, D. & A. Tversky (1979)) based on psychological experiments suggests that personal utility function is defined on deviations from the reference point; generally concave for gains, convex for losses; and steeper for losses than for the gains. Example of such a utility function is drawn on the figure 3. From this figure it could be seen that derivation p^- is lager than p^+. But p^- is increasing which means that borders of the equilibrium interval sometimes could be unstable.

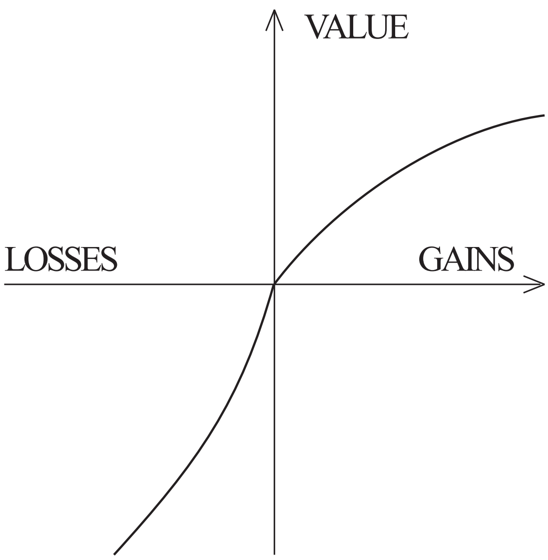

Рис. 3: Utility function

3 Countries on plain

In order to be sure that the unique equilibrium border exists, let us assume in what follows that all countries are governed under the veto rule. Now consider the two-dimensional case where the social welfare of i-th country $W_i = W_i(S_i, L)$ depends on its territory

$$\frac{\partial}{\partial S_i} W_i > 0, \quad \frac{\partial^2}{\partial S_i^2} W_i < 0$$

and on the length if the border L

$$\frac{\partial}{\partial L}(W_1 + W_2) = -\alpha < 0. \tag{8}$$

The letter implies that the expenses for supporting the border are proportional to its length and paid by both countries in a proportion that doesn't matter. If country 1 is

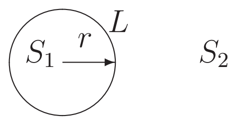

Рис. 4: Circular country surrounded by infinite country

surrounded by country 2, the optimal form of the border at the constant territory S_1 is a circle (fig. 4). This form will be approved by all citizens because it minimizes the expenses on the border's support.

Proposition 2. Under above conditions the equilibrium border is obtained from

$$\text{FOC (stationarity):} \quad p_1 = p_2 + \frac{\alpha}{r}, \tag{9}$$

where r – is the border radius.

Thus, the border is in the equilibrium when the price of land for the inner country p_1 is equal to that for the outer country plus the cost α of the border unit divided by the border radius r.

Formula (9) has a direct analogy in physics with the condition of bubbling of overheated liquid

$$p_1 \geq p_2 + \frac{\alpha}{r^2},$$

where bubble of radius r could exist if the pressure of the gas in the bubble p_1 is not less than pressure outside p_2 plus an additional pressure dew to the surface tension which is inversely proportional to the squared radius of the curvature of the bubble surface. The radius is squared because three-dimensional bubble has a two-dimensional border in contrast with our one-dimensional border case of the country on plain.

4 Example of state equation

Let the individual utility function be

$$u_j = A_j S^k, \qquad j = 1 \dots N,$$

where S is the territory of a group, k is the technological parameter of the group, A_j is the individual utility share of a citizen j, N is the number of people in the group.

Then, marginal welfare will be

$$p = k\, S^{k-1} \sum_{j=1}^{N} A_j = k\, \frac{N}{S} \frac{S^k}{N} \sum_{j=1}^{N} A_j = k\, n\, u, \tag{10}$$

where $n = N/S$ is the people's concentration, $u = \sum u_j/N$ is the average utility in the group. Variables p, n and u could be considered as the state variables of the group.

There is a direct analogy in physics with the state equation of the ideal gas, where p stands for a pressure, n for a concentration of molecules, u for a temperature (mean energy of molecules) and k – Boltzmann constant.

The individual utility share A_j reflects her passionatity, i.e. her preference of the increase of territory to the increase of money. Thus, the higher is passionatity of individuals, the more average utility in the group u is, and the more is the price of its territory.

5 War versus trade

Let us now assume that, instead of paying for the territory, a country could spend same money for hiring the army and conquer the territory. Which of the two alternatives – to buy or to conquer the country – would it prefer?

The same dilemma has been studied before (e. g. Grossman & Mendoza, 2001, 2004) in the economic theory of empire building using examples of the Roman, Mongol, Ottoman, and Nazi German empires, where three strategies where considered: Uncoerced Annexation, Coerced Annexation, and Attempted Conquest.

In order to answer this question, we will use a different game-theoretic model, where a country could cheat, for example take money but not give territory or vice versa. Country 1 has the strategies to buy or to conquer, while country 2 has the strategies to cheat or not to cheat (see figure 5).

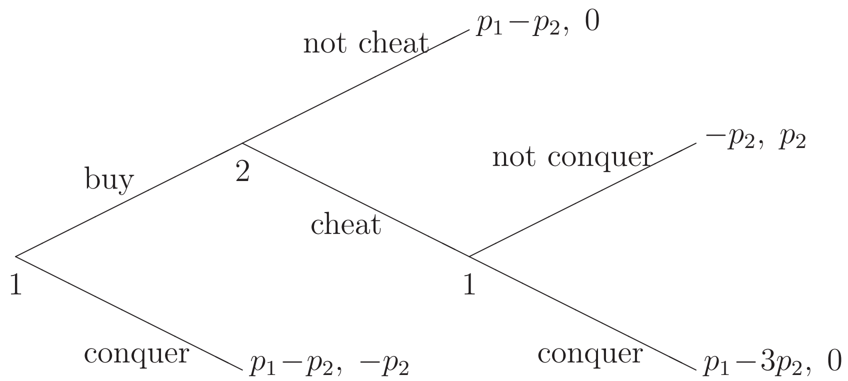

Рис. 5: Game

Let the war process be the same as that of the trade with the only difference that the seller does not get a payment. Country which hires the greatest army wins. The Army could be treated as a third player which can not gain utility from the land itself but only conquers territory for the country which pays the highest price. If country 1 wants to conquer a part of country 2 it pays the army a little bit more than p_1 because otherwise country 2 can conquer territory back by paying p_1. Thus, the conquest looks like an auction.

Let the territory be traded with the minimum seller price p_2, otherwise the conquest is preferable. A country pays the army only if its welfare surplus allows it to win, otherwise it retreats. Hence, countries never fight, one hires the army and occupy a part of the territory of another country or does not hire the army because another could hire not less army itself.

Thus the cheater (country 2) after getting money invests $p_2 + p_2$ in the army to protect from country 1 attack with maximum army cost p_1. Hence, the cheating condition is $2p_2 > p_1$, and it is a Nesh equilibrium in pure strategies in this subgame. The following conclusion is that the countries near the equilibrium (i.e. $p_1 \approx p_2$) will always fight for the territory instead of trading because the threat of cheating (it is the subgame perfect Nash equilibrium, SPNE). Only the country which values territory at least twice as its neighbor can afford itself buying instead of conquering.

6 Mechanisms of migration

Migration between two countries proceeds also with permission from both countries involved and if the individual utility of migrant increases. With the assumption of decreasing personal utility function on the concentration of people, there are two ways of migration. The first way is when a person sells her land in her country and buys land in the country of which citizen she wants to be. Thus, she freely lives her country because her former compatriots get her land at a little bit cheeper price and then she is accepted by voting in new country because she compensates its citizens their loss of personal space buying land at a little bit higher price. The second way of migration is when a part of the country territory where person is living has been conquered or sold. In the second case person has a choice whether to stay in a new country or to leave in her own country, maybe taking compensation from expanding country or just because she wants to live in her native country. The first case is impossible under veto rule because all people get payment sufficient to compensate their loss in utility due to their territory contraction. Thus we will consider the case of migration due to majority rule or conquest.

7 Limits of the country expansion

In this section we will describe the expansion of a circle country surrounded by a big one in which not all citizens are properly compensated for the loss of their land, as it usually is in the case of conquest. In this case some people would prefer to stay on their land and become the citizens of expanding country. For the sake of simplicity let the share of people who prefer to become the citizens of the expanding country be a constant, denote it by K_s. We also suppose that a share K_d of people of the expanding country on the new territory just dies. The coefficients K_s and K_d are exogenous. Thus, the total number N of people in the expanding group will change according to the following law

$$dN = K_s n_0 dS - K_d\, n\, dS, \tag{11}$$

where n is the concentration of people in the country, n_0 is the concentration of people in the surrounding country, and dS is the territory increment of the expanding country. From the expression for the people number $N = S\,n$ we also get $dN = S\,dn + n\,dS$. Substituting the latter into (11) and separating variables, we obtain an integral equation:

$$\int_{n_1}^{n} \frac{dn}{K_s n_0 - (1 + K_d)n} = \int_{S_1}^{S} \frac{dS}{S}, \tag{12}$$

where S_1 is the initial square of the country, n_1 is the initial concentration of people in the country. Taking the integrals in (12) we get: $(Kn_0 - n)/(Kn_0 - n_1) = (S/S_1)^{-(1+K_d)}$, where constant $K = K_s/(1 + K_d)$. Since the country has a form of a circle $S/S_1 = (r/r_2)^2$, after some transformations we get the expression for the concentration of people in the country depending on the radius of its territory $n = Kn_0 + (n_1 - Kn_0)(r_1/r)^{2+2K_d}$. Substituting the latter expression into the equation (10) we get the dependence of a land price inside the expanding country on its radius $p = p_2 + \beta r^{-2(1+K_d)}$, where $p_2 = Kn_0kT$ – the land price at $r \to \infty$, and coefficient $\beta = kT(n_1 - Kn_0)r_1^{2(1+K_d)}$. Substituting the latter expression into (9) we get a condition for country expansion

$$p_2 + \frac{\beta}{r^{2(1+K_d)}} \geq p_0 + \frac{\alpha}{r}. \tag{13}$$

In order to determine the maximal and minimal possible sizes of the country we depict the values of the right L_2 and left L_1 sides of the inequality (13) on the figure 6 for two cases: a) $n_1 > Kn_0$, and b) $n_1 < Kn_0$. When $n_1 = Kn_0$ the curve L_1 degenerates into horizontal line at the level p_2. From the both plots it is clearly seen that for existence of the domain of unlimited expansion the inequality $p_2 > p_0$ is necessary i. e. $Kku > k_0u_0$. It is impossible if the share of staying peoples of surrounding country $K_s = 0$ because it means that $K = 0$. Hence, for the optimal expansion policy it is necessary to increase K

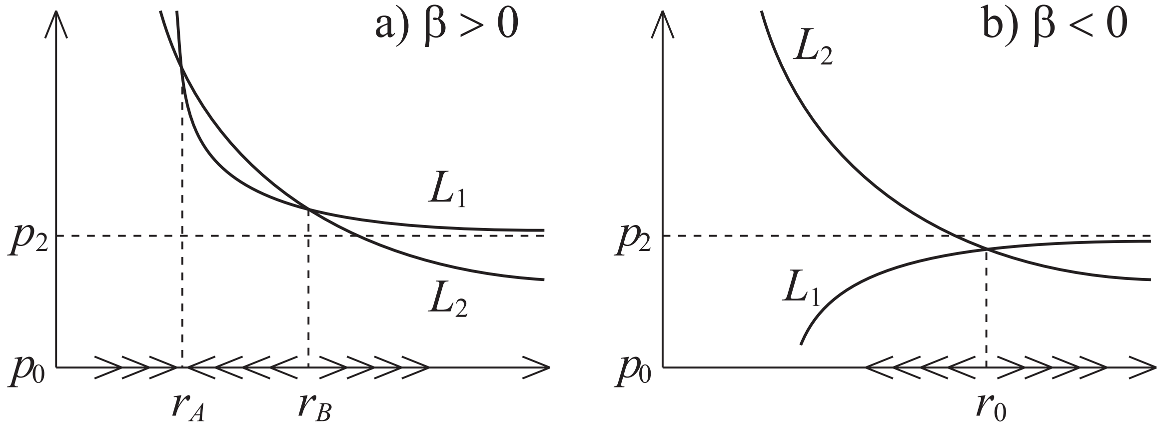

Рис. 6: Left and right sides of the inequality (13)

maximally, i. e. no one of the people from expanding country should die and all people on the occupied territory should join the expanding country.

On the plot a) at the point r_A the country is in the unstable equilibrium, while at the point r_B it is in stable. Because any small negative deviation of the radius from r_B leads to the unlimited expansion while, a positive one to the decreasing of the radius to r_A. At the initial radius $r_1 < r_A$ the expansion of the country is limited by the radius r_A. If the curves L_2 and L_1 not intersect on a) the country expands unlimitedly at any initial radius. On the plot b) there is only the unstable equilibrium point r_0 which is the minimal possible radius of the country. At the grater radius the country unlimitedly expands. If the hyperbolas L_2 and L_1 do not intersect on the plot b) the country can not exist at any size.

In the regime b) the expansion proceeds only because technical k and energetic u superiority of the country in comparison with its surroundings. While in the regime a) group additionally expands because of its superiority in the concentration of people. Nevertheless if we suppose the positive dependence of the expansion velocity on the price difference at the border $L_1 - L_2$ then in the case a) the unlimited expansion proceeds slower than in the case b) at the same initial price difference because in b) the inner price L_1 rises while the radius increase in contract with a).

There were historical examples of expansion in the quick regime b) when the initial people concentration of expanding group is less then that of the surroundings: conquest of the New World by Europeans, and joining of the Siberia to the Russia. The slow regime a) appears to be typical for long term interacting groups which have the same levels of technical development and passionarity because of the technology and people exchange but due to some reasons one group has higher people concentration.

Conclusion

In this work a new mechanism governing the dynamics of territory changing between groups of people such as countries is proposed, based on trading with approval of both sides under particular voting rule (veto rule, majority rule,...). The possible reasons for multiple stable equilibria could be transaction costs and different marginal utility for gains an losses.

Conquest of the territory is viewed as a special case of trading where a “seller” country does not get a payment for its territory. It was found that only the country which values territory at least twice as its neighbor can afford itself buying instead of conquering.

In the two-dimensional case was considered, where the cost of the border between countries is proportional to its length, an influence of the border curvature on the land price has been studied. A state equation of the group was obtained based on particular personal utility function. This equation appeared to be similar to the state equation of ideal gas. Under migration rules the maximal territory expansion and the minimal possible territory are evaluated for a small group surrounded by a big one. Regime b) is similar to the process of bubbling of the overheated liquid.

The possible extension of this work is to make endogenous the share K_S of the people who stays on their land after its conquest.

Appendix

Proof of Proposition 1.

Let $\underline{x} \geq \overline{x}$. Then, from monotonic decreasing property of marginal utilities ($W_1''(x) < 0, W_2''(x) < 0 \; \forall x \in [0,1]$) we have

$$W_1'(\underline{x}) \leq W_1'(\overline{x}) \quad W_2'(\underline{x}) \leq W_2'(\overline{x})$$

substituting here

$$W_1'(\overline{x}) = -\frac{1}{2}W_2'(\overline{x}), \quad W_2'(\underline{x}) = -\frac{1}{2}W_1'(\underline{x})$$

from (3) and (4) and summing we obtain

$$W_1'(\underline{x}) \leq W_2'(\overline{x})$$

which is a contradiction since $W_2'(\overline{x}) < 0$ and $W_1'(\underline{x}) > 0$. Thus $\underline{x} < \overline{x}$.

Proof of Proposition 2.

$$FOC: \quad d(W_1 + W_2) = 0,$$

$$dW_i(S_i, L) = \frac{\partial W_i}{\partial S_i} dS_i + \frac{\partial W_i}{\partial L} dL.$$

Taking into account (5) and (8) after summing we get

$$d(W_1 + W_2) = p_1 \, dS_1 + p_2 \, dS_2 - \alpha dL.$$

From geometry of the problem we have got expressions $dS_2 = -dS_1$ and $dL = \frac{dS_1}{r}$; after substitution and dividing by dS_1 we get $p_1 - p_2 - \frac{\alpha}{r} = 0$ and consequently (9).

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